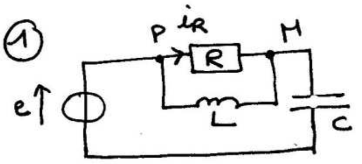
TD Circuits linéaires en régime sinusoïdal forcé - Correction

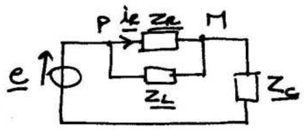
Exercice 4 : Circuit Bouchon

1)

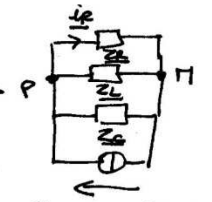


$\longleftrightarrow$

$$\begin{cases} \underline{e} = e_m e^{j\omega t} \\ \underline{i_R} = I_{Rm} e^{j(\omega t+\varphi)} \\ \quad = \underline{I_{Rm}}\, e^{j\omega t} \end{cases}$$



$\longleftrightarrow$



Equivalence de Thévenin-Norton : $\underline{\eta} = \underline{Y_C}\,\underline{e}$

avec $\begin{cases} \underline{Y_C} = j\omega C \\ \underline{Z_L} = j\omega L \\ \underline{Z_R} = R \end{cases}$

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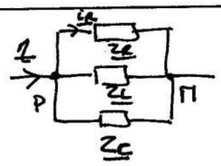
2) Diviseur de courant



$$\underline{i_R} = \frac{\underline{Y_R}}{\underline{Y_R}+\underline{Y_L}+\underline{Y_C}}\,\underline{\eta}$$

Soit $\underline{i_R} = \dfrac{\frac{1}{R}}{\frac{1}{R}+\frac{1}{j\omega L}+j\omega C}\, j\omega C\underline{e} \implies \boxed{\underline{i_R} = \dfrac{j\omega C\,\underline{e}}{1+jR\left(\omega C-\frac{1}{\omega L}\right)}}$

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3) Pour que $\underline{i_R}$ soit indépendant de R il suffit que $\omega C - \dfrac{1}{\omega L} = 0$ soit $\omega = \sqrt{\dfrac{1}{LC}}$

alors $\boxed{\underline{i_R}\left(\omega_0 = \frac{1}{\sqrt{LC}}\right) = j\omega_0 C\underline{e} = \underline{\eta}(\omega_0)}$. A cette pulsation ω_0 le dipôle

(L//C) constitue un circuit bouchon « refusant la pulsation ω_0 ».

En effet d'après la loi des noeuds : $\underline{\eta} = \underline{i_R} + \underbrace{\underline{i_L} + \underline{i_C}}_{=0 \text{ pour } \omega=\omega_0}$.

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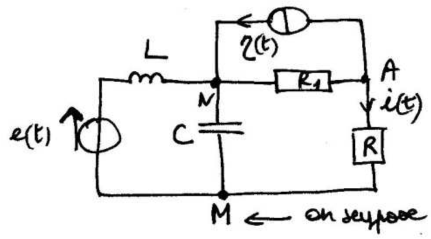
4) • Pour $\omega = \omega_0$, $\underline{i_R} = j\omega_0 C\,\underline{e} \implies |\underline{i_R}| = \omega_0 C\,|\underline{e}| \implies I_{Rm} = \omega_0 C e_m$

$I_{Rm} = 10^2 . 10^2 \pi . 10^{-7}$ A $\implies$ $I_{Rm} = 3$ mA

• $\underline{I_{Rm}} = j\omega_0 C e_m \implies \arg(\underline{I_{Rm}}) = \varphi = \dfrac{\pi}{2}$

Exercice 5 : Recherche d'un courant par le théorème de Millman



$$\begin{cases} e(t) = e_m \cos(\omega t) \\ \eta(t) = \eta_m \cos(\omega t) \\ i(t) = I_m \cos(\omega t + \varphi') \end{cases} \longrightarrow \begin{cases} \underline{e} = e_m e^{j\omega t} \\ \underline{\eta} = \eta_m e^{j\omega t} \\ \underline{i} = I_m e^{j(\omega t + \varphi')} = \underline{I_m} e^{j\omega t} \end{cases}$$

M ← on suppose qu'il s'agit de la masse du circuit $\underline{V_M} = 0$.

(1) On applique le théorème de Millman aux noeuds :

* $\boxed{N}$ $$\underline{V_N} = \frac{\underline{Y_L}\,\underline{e} + \underline{Y_{R_1}}\,\underline{V_A} + \underline{\eta}}{\underline{Y_L} + \underline{Y_C} + \underline{Y_{R_1}}} \quad (1)$$

avec $$\begin{cases} \underline{Y_L} = \dfrac{1}{j\omega L} \\ \underline{Y_C} = j\omega C \\ \underline{Y_R} = \dfrac{1}{R} \end{cases}$$

* $\boxed{A}$ $$\underline{V_A} = \frac{\underline{V_N}\,\underline{Y_{R_1}} - \underline{\eta}}{\underline{Y_R} + \underline{Y_{R_1}}} \quad (2)$$

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(2) • $\underline{V_A} - \underline{V_M} = \underline{V_A} = R\,\underline{i}$ d'où $\underline{i} = \dfrac{\underline{V_A}}{R}$

D'après (2) $$\underline{V_N} = \underline{V_A}\left(\frac{\underline{Y_R}}{\underline{Y_{R_1}}} + 1\right) + \frac{\underline{\eta}}{\underline{Y_{R_1}}}$$

D'après (1) $$\frac{\underline{Y_L}\,\underline{e} + \underline{Y_{R_1}}\,\underline{V_A} + \underline{\eta}}{\underline{Y_L} + \underline{Y_C} + \underline{Y_{R_1}}} = \underline{V_A}\left(\frac{\underline{Y_R}}{\underline{Y_{R_1}}} + 1\right) + \frac{\underline{\eta}}{\underline{Y_{R_1}}}$$

$$\Rightarrow \underline{V_A}\left[\underline{Y_{R_1}} - \left(\frac{\underline{Y_R}}{\underline{Y_{R_1}}} + 1\right)\left(\underline{Y_L} + \underline{Y_C} + \underline{Y_{R_1}}\right)\right] = -\underline{Y_L}\,\underline{e} - \underline{\eta} + \frac{\underline{\eta}}{\underline{Y_{R_1}}}\left(\underline{Y_L} + \underline{Y_C} + \underline{Y_{R_1}}\right)$$

$$\Rightarrow \underline{V_A}\left[-\underline{Y_R} - \left(\frac{\underline{Y_R}}{\underline{Y_{R_1}}} + 1\right)\left(\underline{Y_L} + \underline{Y_C}\right)\right] = -\underline{Y_L}\,\underline{e} + \frac{\underline{\eta}}{\underline{Y_{R_1}}}\left(\underline{Y_L} + \underline{Y_C}\right)$$

$\times -\dfrac{1}{\underline{Y_L}}$

$$\Rightarrow \underline{V_A}\left[\frac{\underline{Y_R}}{\underline{Y_L}} + \left(\frac{\underline{Y_R}}{\underline{Y_{R_1}}} + 1\right)\left(1 + \frac{\underline{Y_C}}{\underline{Y_L}}\right)\right] = \underline{e} - \frac{\underline{\eta}}{\underline{Y_{R_1}}}\left(1 + \frac{\underline{Y_C}}{\underline{Y_L}}\right)$$

$$\Rightarrow \underline{Y_R}\,\underline{V_A}\left[\frac{1}{\underline{Y_L}} + \left(\frac{1}{\underline{Y_{R_1}}} + \frac{1}{\underline{Y_R}}\right)\left(1 + \frac{\underline{Y_C}}{\underline{Y_L}}\right)\right] = \underline{e} - \frac{\underline{\eta}}{\underline{Y_{R_1}}}\left(1 + \frac{\underline{Y_C}}{\underline{Y_L}}\right)$$

En remplaçant les admittances par leur valeur il vient :

$$\frac{\underline{V_A}}{R}\left[j\omega L + (R_1+R)\left(1+(j\omega C)(j\omega L)\right)\right] = \underline{e} - \eta R_1\left(1+(j\omega C)(j\omega L)\right)$$

$$\Rightarrow \boxed{\underline{i} = \frac{\underline{V_A}}{R} = \frac{\underline{e} - \eta R_1(1-\omega^2 LC)}{(R_1+R)(1-\omega^2 LC) + j\omega L}}$$

On peut écrire $\quad \underline{I_m} = \dfrac{e_m - \eta_m R_1(1-\omega^2 LC)}{(R_1+R)(1-\omega^2 LC) + j\omega L}$

$$I_m = |\underline{I_m}| = \frac{e_m - \eta_m R_1(1-\omega^2 LC)}{\sqrt{\left[(R_1+R)(1-\omega^2 LC)\right]^2 + (\omega L)^2}} > 0 \ (\text{ok})$$

$$\begin{cases} \varphi' = \arg(\underline{I_m}) = -\arg\left((R_1+R)(1-\omega^2 LC) + j\omega L\right) \\ \tan\varphi' = -\dfrac{\omega L}{(R_1+R)(1-\omega^2 LC)} \end{cases}$$